Tutorial 1

1   Set Theory

Question 1:

a. Given $U = [-5, 5], \quad A = [-1, 1], \quad B = [5, 8], \quad C = [1, 3], \quad D = (2, 4)$.

Find $A^C, \quad D', \quad A \cap B, \quad B \cup U, \quad \emptyset^C, \quad A' \cap D'$.

b. Let $A = \{2, 3, 4\}, \quad B = \{2, 5, 6\}, \quad C = \{5, 6, 2\}$, and $D = \{6\}$ Which of the following statements are true?

a. $4 \in C$	b. $5 \in C$	c. $A \subseteq B$
d. $D \subseteq C$	e. $B = C$	f. $A \cap B = \{2, 3, 4, 5, 6\}$

Question 2:

Find the cartesian product of the following sets. Give a graphical and analytical solution.

a. $A = [3, 5], \quad B = [5, 8)$

b. $A = \{1, 2, 3\}, \quad B = \{1, 5\}$

Question 3:

Give an explicit list of the elements of each of the following sets:

a. $E = \{x \mid x \in \mathbb{N}, -2 < x < 19\}$

b. $K = \{k \mid k \in \mathbb{Z}, k \notin \mathbb{N}, k < 9\}$

c. $A = \{3n + 2 : n \in \mathbb{N}, n \leq 10\}$

2   Matrix Algebra

Question 1:

Find $A + B, \quad A - B, \quad 2A, \quad AB, \quad BA, \quad A^2$ for the following matrices:

$$A = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

Question 2:

Compute $AB, \quad BA, \quad A^\top, \quad B'$ for:

$$A = \begin{pmatrix} 0 & 1 & 3 \\ 3 & 4 & -1 \\ 4 & 2 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 2 \\ 1 & 0 \\ -1 & 1 \end{pmatrix}$$

Question 3:

What is the determinant of the following matrices?

a. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

b. $\begin{pmatrix} 25 & 8 \\ -12 & 4 \end{pmatrix}$

c. $\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 7 & 11 \end{pmatrix}$

d. $\begin{pmatrix} 5 & 2 & 5 \\ 3 & 7 & 8 \\ 1 & 2 & 6 \end{pmatrix}$

Question 4:

Find inverse of the following matrices and verify that $A^{-1} \cdot A = A \cdot A^{-1} = I$

a. $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

b. $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$

c. $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

d. $\begin{pmatrix} 8 & 2 & 3 \\ 4 & 2 & 0 \\ 6 & 5 & 2 \end{pmatrix}$

Question 5:

Solve the following system of equations:

a.
$$\begin{cases} 8x_1 + 2x_2 + 3x_3 = 18 \\ 4x_1 + 2x_2 = 8 \\ 6x_1 + 5x_2 + 2x_3 = 20 \end{cases}$$

b.
$$\begin{cases} 2x_1 + x_2 + x_3 = 40 \\ x_1 + 2x_2 + 3x_3 = 60 \\ 2x_1 - x_2 + x_3 = 20 \end{cases}$$

c. $Xq = p$, where $X = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 3 & 6 \end{pmatrix}$ and $p = \begin{pmatrix} 2.2 \\ 3.5 \end{pmatrix}$

d.
$$\begin{cases} 2a + b + c = 40 \\ 6a + 3b + 3c = 60 \end{cases}$$

Question 6:

$$u = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \quad v = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

a. Let $0 \leq c, d \leq 1$ and $c + d = 1$. Draw $cu + dv$.

b. Let $c, d \in \mathbb{R}$ and $c + d = 1$. Draw $cu + dv$.

c. Let $0 \leq c, d \leq 1$. Draw $cu + dv$.

d. Let $c, d \geq 0$. Draw $cu + dv$.