

Tutorial 2

1 Functions

Question 1:

Answer the following questions for functions described below:

What are the domain and the range of f ?

What is the value of f at the values given in brackets?

Draw a plot of f .

What type of function is f ?

Is it continuous? Is it monotonic?

- $f : [0, 2] \rightarrow \mathbb{R}, \quad f(x) = x^2 - 2x + 1, \quad (0, 1, 2)$
- $f : \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = 2x - 2, \quad (-2, 0, 1, 2, 4)$
- $f : \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = e^x, \quad (-1, 0, 1, 2, 3)$ use calculator!
- $f : \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = e^{2x}, \quad (-1, 0, 1, 2, 3)$ use calculator. Plot it together with the previous graph.
- $f : \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = \ln x, \quad (1, 2, 3)$ use calculator!
- $f : \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = 2\ln x, \quad (1, 2, 3)$ use calculator. Plot it together with the previous graph.

Question 2:

Simplify/reformulate the following expressions:

- $\log_a a^3 + \log_a b^2$
- $\ln e^x + 2x \ln e$
- $\log \left(\frac{a^2}{b^6} \right)$
- $\log_3 81^2$
- $\log_2 4^0$
- $\ln (AK^\alpha L^{1-\beta})$

Question 3:

Compute the following limits:

a. $\lim_{x \rightarrow -3} x^2 - 9x + 3$

b. $\lim_{x \rightarrow 2} \frac{1}{(x-2)^2}$

c. $\lim_{x \rightarrow 1} \frac{2 \ln x}{x-1}$

d. $\lim_{x \rightarrow \infty} \frac{x^4 + 3x^3 + x + 7}{5x^4 + 7x^3}$

e. $\lim_{x \rightarrow \infty} \frac{x^7 + 3x^3 + x^2}{8x^4 + 6x^3}$

f. $\lim_{x \rightarrow 4^-} \frac{1}{x-4}$

2 Derivatives

Question 1:

Calculate derivatives of the following functions with respect to x .

a. $f(x) = 3x^5 + 4x^2 + 2x + 10$

b. $f(x) = 2^x$

c. $f(x) = [8x^2 + 2x^4 + x + 2x^3]^3$

d. $f(x) = 2^x e^x$

e. $f(x) = \sqrt{x^2 + 2}$

f. $f(x) = x^2 \log(x^3 + 2x + 1)$

g. $f(x) = [x^5 + 3x^3 + x + 7]^{\frac{3}{2}}$

h. $f(x) = \frac{2x + 1}{\sqrt{x^3 + 4}}$

Question 2:

Calculate the derivatives w.r.t. the variable in brackets. Take all the other variables as constant.

a. $c\sqrt{y} + d \ln z$ (y)

b. $\frac{aq^2 \ln c + bq^3 e^d}{aq^8 - b}$ (q)

c. $z^8 + y^4 + z^2 y^3 + zy + z^2$ (u)

d. $a \ln r + \ln(sr^2)(s^2 + s + 3)$ (r)

Question 3:

- a. What is elasticity of a function (demand or production for example)? Give a mathematical definition and economical interpretation.

- b. Given the linear demand function $q(p) = a - bp$, find the elasticity of demand with respect to price p .
- c. Given the production function $Y = AK^\alpha L^{1-\alpha}$, where L is labor, K is capital, and $A \in \mathbb{R}$. Find the elasticity of substitution of output with respect to labor and capital.

Question 4:

Find the rate of growth for the following functions:

- a. $f(t) = e^{\sqrt{3t}}$
- b. $f(t) = e^t t^2$
- c. $f(t) = \frac{t^2 + 1}{e^{3t} + 7}$

Question 5:

Find Taylor approximation (of order 3) of function $g(x) = \log(1 + x)$ at point $x = 0$.

3 Integrals

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| 1. $\int (3x^2 + 5x + 2) dx$ | 2. $\int e^2 x dx$ |
| 3. $\int (3x^6 - 2x) dx$ | 4. $\int \frac{2x+4x^3}{x^2+x^4+3} dx$ |
| 5. $8 \int \frac{(\ln x)^2}{x} dx$ | 6. $\int 6^{5x} dx$ |
| 7. $\int x e^x dx$ | 8. $\int z^2 e^z dz$ |
| 9. $\int_0^1 \int_0^1 \frac{2}{5} (2x + 2y) dx dy$ | 10. $\int \int_{x^2+y^2 \leq R} 1 dx dy$ |
| 11. $\int_0^z y^2 dy$ | 12. $\int_0^1 \int_0^{1+u} 1 dv du$ |